Score:

Name: Solutions
Period (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

SM 261 – Matrix Algebra – Quiz 21 – Open Notes
Section 5.3 – Diagonalization

1. Diagonalize the matrix: $A = \begin{bmatrix} 3 & -2 \\ 4 & -3 \end{bmatrix}$ (i.e. put in form $A = PDP^{-1}$).

Find Eigenvalues: $\det \begin{bmatrix} 3-\lambda & -2 \\ 4 & -3-\lambda \end{bmatrix} = 0 \Rightarrow -9 - 3\lambda + 3\lambda + \lambda^2 + 8 = 0$

$\Rightarrow (\lambda^2 - 1) = 0 \qquad \lambda = +1, -1$

$\lambda = 1$ $\begin{bmatrix} 2 & -2 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow x_1 = x_2 \Rightarrow V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = -1 \Rightarrow \begin{bmatrix} 4 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow 4x_1 = 2x_2 \Rightarrow 2x_1 = x_2 \Rightarrow \vec{V}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$P \qquad D \qquad P^{-1}$ ← used cramers rule

$$\therefore \boxed{A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}}$$

2. Using diagonalized form, find A^{25}.

$A^{25} = PD^{25}P^{-1} \Rightarrow D^{25} = \begin{bmatrix} 1^{25} & 0 \\ 0 & -1^{25} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

This is just D!

$\therefore A^{25} = A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$!!!

This Also Implies that A is its own inverse!