

Score:

 Name: Solutions
 Period (circle one): 1 2 3 4 5 6
 Team (circle one): a b c d e f

SM 261 – Matrix Algebra – Quiz 23

Section 5.5 – Dynamical Systems

1. The population of a herd of buffalo is classified into the following groups: Adults (A) and calves (C). Every year each adult buffalo produces 1.2 offspring. Every year 90% of the adult population survives to live another year and 60% of the calf population passes into adulthood (the remaining 40% are removed from the population i.e. they die ☹).

Write a matrix equation that describes the population dynamics. Use eigenvalue decomposition to determine the fate of the buffalo.

$$\begin{array}{l} A_{k+1} = .9A_k + .6C_k \\ C_{k+1} = 1.2A_k + 0C_k \end{array} \Rightarrow \begin{bmatrix} A_{k+1} \\ C_{k+1} \end{bmatrix} = \begin{bmatrix} .9 & .6 \\ 1.2 & 0 \end{bmatrix} \begin{bmatrix} A_k \\ C_k \end{bmatrix}$$

$$\Rightarrow (.9 - \lambda)(-\lambda) - .72 = 0 \Rightarrow \lambda^2 - .9\lambda - .72 = 0$$

$$\lambda = \frac{.9 \pm \sqrt{(.9)^2 - 4(-.72)}}{2} = \begin{matrix} 1.4105, & -.5105 \\ \lambda_1 & \lambda_2 \end{matrix}$$

$$\Rightarrow \vec{v}_1 \Rightarrow \begin{bmatrix} .9 - 1.4105 & .6 \\ 1.2 & -1.4105 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} -.5105 & .6 \\ 1.2 & -1.4105 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\Rightarrow -.5105x_1 = .6x_2 \Rightarrow x_1 = \frac{.6}{(.5105)}x_2 \Rightarrow x_1 = 1.1753x_2$$

$$\Rightarrow \left(\lambda_1 = 1.4105 \quad \vec{v}_1 = \begin{bmatrix} 1.1753 \\ 1 \end{bmatrix} \right)$$

$$\Rightarrow \vec{v}_2 \Rightarrow \begin{bmatrix} 1.4105 & .6 \\ 1.2 & .5105 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow x_1 = \frac{-.6x_2}{1.4105} \Rightarrow x_1 = -.4254x_2$$

$$\Rightarrow \left(\vec{v}_2 = \begin{bmatrix} -.4254 \\ 1 \end{bmatrix} \quad \lambda_2 = -.5105 \right)$$

$$\begin{matrix} \circ \\ \circ \circ \end{matrix} \begin{bmatrix} A_k \\ C_k \end{bmatrix} = C_1 (1.41105)^k \begin{bmatrix} 1.1753 \\ 1 \end{bmatrix} + C_2 (-0.5105)^k \begin{bmatrix} -1.4251 \\ 1 \end{bmatrix}$$

as k gets big
this term goes
away

This term dominates & is > 1

\therefore Population will grow!!!