

Score:

Name: Solutions  
Period (circle one): 1 2 3 4 5 6  
Team (circle one): a b c d e f

**SM 261 – Matrix Algebra – Quiz 25**  
**Section 6.3 – Orthogonal Projections**

1.  $W$  is a subspace of  $\mathbf{R}^4$  and is has an orthogonal basis  $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ .  $\vec{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ .

a. Find  $\text{proj}_W \vec{y}$ .

$$c_1 = \frac{\vec{y} \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1} = \frac{4}{4} = 1, \quad c_2 = \frac{\vec{y} \cdot \vec{v}_2}{\vec{v}_2 \cdot \vec{v}_2} = \frac{8}{4} = 2$$

$$\vec{y} = (1) \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 1 \\ 3 \end{bmatrix}$$

b. Find  $\text{proj}_{W^\perp} \vec{y}$ .

$$\text{proj}_{W^\perp} \vec{y} = \vec{y} - \text{proj}_W \vec{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} -1 \\ 3 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 2 \\ 1 \end{bmatrix}$$

c. What is  $\text{proj}_W \vec{y} \cdot \text{proj}_{W^\perp} \vec{y}$ ? Should = 0

check:

$$(-1)(2) + (3)(-1) + (1)(2) + (3)(1)$$

$$-2 - 3 + 2 + 3 = 0 \quad \checkmark$$