

Score:

Name: Solutions
 Period (circle one): 1 2 3 4 5 6
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SM 261 – Matrix Algebra – Quiz 26
Section 6.4 – Orthonormal Bases

1. Convert the following basis into an orthonormal basis $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} \right\}$.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\vec{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} - \frac{[1 \ 1 \ -1 \ 1] \cdot [-1 \ 1 \ 1 \ -1]}{\| [1 \ 1 \ -1 \ 1] \|^2} \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} - \frac{-2}{4} \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 3/2 \\ 1/2 \\ -1/2 \end{bmatrix} = \vec{v}_2$$

$$\|\vec{v}_2\| = \left(\frac{1}{4}(1+9+1+1) \right)^{1/2} = \left(\frac{12}{4} \right)^{1/2} = \sqrt{3}$$

$$\vec{u}_1 = \frac{\vec{v}_1}{\|\vec{v}_1\|} = \begin{bmatrix} 1/4 \\ 1/4 \\ -1/4 \\ 1/4 \end{bmatrix}$$

$$\vec{u}_2 = \frac{\vec{v}_2}{\|\vec{v}_2\|} = \begin{bmatrix} -1/2\sqrt{3} \\ 3/2\sqrt{3} \\ 1/2\sqrt{3} \\ -1/2\sqrt{3} \end{bmatrix} = \begin{bmatrix} -\sqrt{3}/6 \\ \sqrt{3}/2 \\ \sqrt{3}/6 \\ -\sqrt{3}/6 \end{bmatrix}$$

$$\Rightarrow B = \left\{ \vec{u}_1, \vec{u}_2 \right\}$$