Score:

Name:_______________________________

Section (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

SM261 – Test #2 Group Test – Fall 2009

Open Book. Due: Wednesday 28 October 2009

- Each Group will turn in one Test.
- No one person in the group shall write up more than one proof.
- If your group has only three members, then no one person in the group shall write up more than two proofs.
- The name of the individual writing the proof shall appear on the top of each page.
- Each proof shall be written up in the following format:

Given: (1) Everything you know from the problem statement.
(2) …….
(3) …….

Proof: (1) Every statement should be supported by a theorem, definition, or justification for an operation.
(2) …..
(3) …..

Q.E.D: quod erat demonstrandum which translates into "which was to be demonstrated.

- **Write Neatly!!!!!!! No points will be granted if I can't read the proof.**
- **This exercise will make up 20% of the test grade.**
- **One of these proofs will appear on Friday's test.**

No marks on this table	
1 (5 pts)	
2 (5 pts)	
3 (5 pts)	
4 (5 pts)	

Name:______________________________

1. If det(A)=0 then the inverse of A does not exist.

Given: det(A)=0.

Prove: A^{-1} does not exist.

1. A can be converted to echelon form by applying row swaps or adding a multiple of one row to another, i.e.

$$\begin{bmatrix} \mathbf{a} & * & * & * \\ \mathbf{0} & \mathbf{b} & * & * \\ \mathbf{0} & \mathbf{0} & \ldots & * \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{n} \end{bmatrix}$$

2. Thus det(A) is $\pm$ the product of the diagonal elements i.e. $\pm$(a)(b)…(n) (**Theorem 3.2)**

3. If det(A)=0 then one of the diagonal elements must be equal to 0.

4. Therefore A has less than n-pivots.

5. Therefore A is not invertible. (**Theorem 2.8).**

QED

Name:________________________________

2. Prove algebraically (vice geometrically) that the line ax+by=0 is a subspace of $\mathbb{R}^2$.

Given: The line ax+by=0.

Prove: ax+by =0 is a subspace of $\mathbb{R}^2$.

1. The zero vector $\begin{bmatrix}0\\0\end{bmatrix}$ is in the space ax+by=0.
2. Solve for y, i.e y=-(a/b)x. Therefore a basis vector for the line is $\vec{b} = \begin{bmatrix}1\\-a/b\end{bmatrix}$, i.e. dimension of space is 1.
3. Let $\vec{v} = \begin{bmatrix}1\\-a/b\end{bmatrix}$, then $\overrightarrow{kv} = k\begin{bmatrix}1\\-a/b\end{bmatrix}$, is on the line (i.e. the space is closed under scalar multiplication).
4. Let $\overrightarrow{u_1} = k\begin{bmatrix}1\\-a/b\end{bmatrix}$ and $\overrightarrow{u_2} = h\begin{bmatrix}1\\-a/b\end{bmatrix}$, i.e. both vectors in the space.
5. Then $\overrightarrow{u_1} + \overrightarrow{u_2} = (k+h)\begin{bmatrix}1\\-a/b\end{bmatrix}$ is on the line (i.e the space is closed under vector addition).
6. Therefore ax+by=0 is a subspace of $\mathbb{R}^2$. (**Definition of subspace, p. 220**)

QED

Name:________________________________

3. If A is an $n \times n$ matrix, prove that dim(Col A)+dim(Nul A)=dim($\mathbb{R}^n$).

Given: A is an $n \times n$ matrix.

Prove: dim(Col A)+dim(Nul A)=dim($\mathbb{R}^n$).

1. dim($\mathbb{R}^n$) $= n$.
2. If we convert A to the reduced row echelon form, the resulting matrix will have *n*-*k* pivots where $n - k \geq 0$.
3. Therefore matrix A has *n*-*k* independent columns which form the basis for Col A.
4. Thus dim(Col A)=*n*-*k*
5. When solving for the basis of the null space, the pivot positions mark the dependent variables.
6. Therefore there are *n*-*k* dependent variables, or *k* independent variables.
7. Each independent variable results in a basis vector for the null space. This dim(Nul A)=*k*.
8. Therefore dim(Col A)+dim(Nul A)=n=dim($\mathbb{R}^n$).

QED

Name:______________________________

4. Consider the matrix $A=\begin{bmatrix} a & b & c \\ d & e & f \\ h & i & j \end{bmatrix}$. Prove that adding any multiple of row one to row two does not affect the value det(A).

   a. Let $B=\begin{bmatrix} \mathrm{a} & \mathrm{b} & \mathrm{c} \\ \mathrm{ka+d} & \mathrm{kb+e} & \mathrm{kc+f} \\ \mathrm{h} & \mathrm{i} & \mathrm{j} \end{bmatrix}$

   b. $\det(\mathrm{B}) = \mathrm{a}\begin{vmatrix} \mathrm{kb+e} & \mathrm{kc+f} \\ \mathrm{i} & \mathrm{j} \end{vmatrix} - \mathrm{b}\begin{vmatrix} \mathrm{ka+d} & \mathrm{kc+f} \\ \mathrm{h} & \mathrm{j} \end{vmatrix} + \mathrm{c}\begin{vmatrix} \mathrm{ka+d} & \mathrm{kb+e} \\ \mathrm{h} & \mathrm{i} \end{vmatrix}$

   c. $\longrightarrow \det(\mathrm{B}) = \mathrm{a(kbj + ej - kci - fi) - b(kaj + dj - kch - fh) + c(kai + di - kbh{-}eh}$

   d. $\longrightarrow \det(\mathrm{B}) = \mathrm{akbj + aej - akci - afi - bkaj - bdj + bkch + bfh + ckai + cdi - ckbh - ceh}$

   e. After cancelations: $\det(\mathrm{B}) = \mathrm{aej - afi - bdj + bfh + cdi - ceh}$

   f. But this is just det(A)

   g. Therefore det(B)=det(A)

   h. Hence the roe operation did not change the value of the determinant.

   i. QED

Name:______________________________

5.

Name:______________________________

Suppose vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ span $\Re^4$ and that $T(\vec{v}_1) = T(\vec{v}_2) = T(\vec{v}_3) = T(\vec{v}_4) = 0$.
Prove that T ***is the zero transformation, that is*** $T(\vec{x}) = 0$ ***for all*** $x \in \Re^4$**.**