Score:

Name: Solutions

Section (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

SM261 –Test #2– Fall 2009

1. Find the determinant of $\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & 6 & 7 & 5 \\ 5 & 0 & 4 & 4 \end{bmatrix}$

$$-3\begin{vmatrix} 1 & 0 & 2 \\ 2 & 6 & 5 \\ 5 & 0 & 4 \end{vmatrix} \to -3(6)\begin{vmatrix} 1 & 2 \\ 5 & 4 \end{vmatrix} = -3(6)(4-10)$$

$$= -3(6)(-6) = 108$$

2. If the determinant of $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = k$, then what is the determinant of:

a. $\begin{bmatrix} a & b \\ kc & kd \end{bmatrix} = k(k) = k^2$

b. $\begin{bmatrix} c & d \\ a & b \end{bmatrix} = -k$

c. $\begin{bmatrix} a & b \\ ka+c & kb+d \end{bmatrix} = k$

d. $\begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix} = k^3$

e. $\begin{bmatrix} ka & kb \\ ka+c & kb+d \end{bmatrix} = k$

Prob	Pts	Score
1	10	
2	10	
3	20	
4	10	
5	5 ~~10~~	
6	5 ~~10~~	
7	10	
8	10	
GT	20	
Σ	100	

3. Use Cramer's rule to find the solution to $\begin{cases} 6x + 5y = 2 \\ 8x + 7y = 5 \end{cases}$.

$$\begin{bmatrix} 6 & 5 \\ 8 & 7 \end{bmatrix} \vec{x} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$\text{Det} \begin{bmatrix} 6 & 5 \\ 8 & 7 \end{bmatrix} = (6)(7) - (5)(8) = 2$$

$$\text{Det} \begin{bmatrix} 2 & 5 \\ 5 & 7 \end{bmatrix} = -11 \Rightarrow \boxed{x = \frac{-11}{2}}$$

$$\text{Det} \begin{bmatrix} 6 & 2 \\ 8 & 5 \end{bmatrix} = 14 \Rightarrow \boxed{y = \frac{14}{2} = 7}$$

Check

$$6\left(\frac{-11}{2}\right) + 5(7) = -33 + 35 = 2 \checkmark\checkmark$$

$$8\left(\frac{-11}{2}\right) + 7(7) = -44 + 49 = 5$$

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For the next three problems, let $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

4. Find the basis for:

   a. Col A

$$\text{RREF}(A) = \begin{bmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

pivots

$$\text{Basis Col } A = \left\{ \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} \right\}$$

   b. Nul A

$$x_1 - 2x_2 - x_4 + 3x_5 = 0$$
$$x_3 + 2x_4 - 2x_5 = 0$$

$$x_1 = 2x_2 + x_4 - 3x_5$$
$$x_2 = x_2$$
$$x_3 = -2x_4 + 2x_5$$
$$x_4 = x_4$$
$$x_5 = x_5$$

$$\vec{x} = \underbrace{x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}}_{\text{Basis of Nul } A}$$

5. Determine the following:

   a. Dim Col A $= 2$

   b. Dim Nul A $= 3$

   c. Dim Row A $= 2$

   d. Rank A $= 2$

   e. Rank A^T $= 3$

   f. Col A $\in \Re^n \rightarrow n =$ 3

   g. Nul A $\in \Re^n \rightarrow n =$ 5

   h. Row A $\in \Re^n \rightarrow n =$ 5

6. If A is a 3×5 matrix, determine the following:

   a. Maximum Dim Col A 3

   b. Maximum Dim Row A 3

   c. Minimum Dim Nul A 2

   d. Rank A + Dim Nul A 5

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7. On any given day, a student is either healthy or ill. Of the students who are healthy today, 95% will be healthy tomorrow. Of the students who are ill today, 55% will be ill tomorrow.
   a. What is the stochastic matrix of this situation?
   b. After a long period of time what is the percentage of healthy and ill students (i.e. what are the steady state percentages)?

$$M = \begin{array}{cc} S & H \\ \left[\begin{array}{cc} .95 & .45 \\ .05 & .55 \end{array}\right] \end{array} \begin{array}{c} S \\ H \end{array}$$

$$\Rightarrow M\vec{x} = \vec{x} \Rightarrow (M - I)\vec{x} = 0$$

$$\Rightarrow \begin{bmatrix} -.05 & .45 \\ .05 & -.45 \end{bmatrix} \vec{x} = 0$$

$$\Rightarrow \begin{bmatrix} 1 & -9 \\ 0 & 0 \end{bmatrix} \vec{x} = 0 \Rightarrow x_1 = 9x_2$$

$$\Rightarrow x_1 + x_2 = 100\% \Rightarrow 10x_2 = 100\%$$

$$\Rightarrow x_2 = 10\% = .1 \Rightarrow x_1 = .9$$

Check $\begin{bmatrix} .95 & .45 \\ .05 & .55 \end{bmatrix}\begin{bmatrix} .9 \\ .1 \end{bmatrix} = \begin{bmatrix} .9 \\ .1 \end{bmatrix}$ ✓

8. (10 pts) If A is a 2×2 matrix, and B is created by adding a multiple of row one in A to row two in A, prove algebraically that det A = det B.

(1) let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \det A = ad - bc$

(2) let $B = \begin{bmatrix} a & b \\ ka+c & kb+d \end{bmatrix} \Rightarrow \det B = kab + ad - kab - bc$

$= ad - bc$

(3) $\therefore \det A = \det B$

QED

9. (10 pts) If the basis for V is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ find the coordinates for the vector $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$

$$\text{RREF} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix}$$

coordinates